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RESEARCH ARTICLE

Note on Double Aboodh Transform of Fractional Order and its Properties

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Abstract

In this study, we introduce definitions of a fractional double Aboodh transform of order α, where $\alpha \in [0,1]$, for functions which are fractional differentiable. We then establish some main properties of this transform. Furthermore, we prove some related theorems.

Keywords: Fractional Laplace transform, Summudu transform, double Aboodh transform, Mittag leffler function

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1 Introduction

Integral transform method is one of the most known method for solving partial differential equations of integer and fractional order. In literature, there are various integral transforms like Fourier [1], Natural[2,9], Laplace [3,4], Summdu [5,6], Aboodh [7,8] and so on. These transforms are used in physics, statistics and also in engineering. Double Laplace transform, double Summdu transform and double Aboodh transform were used to solve differential,integral equation and system of differential equation. Aboodh transform first introduce by Khalid Aboodh in 2013 is a transform derived from Fourier integral which is an efficient method to solve partial differential equations. Aboodh introduced a higher version called double Aboodh transform to solve linear partial, fractional and partial integro-differential equations. S. Alfaqih, T. Ozis in 2019 [10]introduced the first Aboodh transform of fractional order and its properties [10]. The objective of the present Article is to define the fractional Double Aboodh transform and to present several main properties and theorems.

This study has been organized as follows: In section 2, we recall some definitions and results related to double Aboodh transform, in section 3, we introduce the definition of fractional double Aboodh transform. In section 4, we present and prove some properties of fractional double Aboodh transform, in section 5; we state the convolution theorem of the fractional double Aboodh transform and its proof. The inversion formula and inversion theorem and its proof of the fractional double Aboodh transform are given in section 6 and 7. Finally, the conclusion follows in section 8.

2 Preliminaries

In this section, we recall the definitions of double Aboodh transform, first fractional Aboodh transform, and the fractional derivative via fractional difference.

Definition 2.1. *let f be acontinuos function of two variables , then the double Aboodh transform of $f(x,t)$ is defined by:*

$$A_{xt}[f(x,t)] = k(p,q) = \frac{1}{pq}\int_0^\infty\int_0^\infty e^{-px}e^{-qt}f(x,t)\,dxdt, x,t>0. \tag{1}$$

And,

$$f(x,t) = A_{xt}^{-1}[k(p,k)] = \frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty} pe^{px}\left[\frac{1}{2\pi i}\int_{\mu-i\infty}^{\mu+i\infty} qe^{qt}k(p,q)\,dq\right]dp, \tag{2}$$

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is the inverse of double Aboodh transform . For more details see [16].

Definition 2.2 (Fractional Derivative via Fractional Difference). *Let $f(x)$ be a continuous function operator $FW(h)$ as following:*

$$\Delta^{\alpha} f(x) = (FW-h)^{\alpha} f(x) = \sum_{j=0}^{\infty} (-1)^j \begin{pmatrix} \alpha \\ j \end{pmatrix} f(\gamma + (\alpha - k)h), \tag{3}$$

where h is apositive real number and denote aconstant discretization span. Then the fractional difference of order $\alpha, \alpha < 1$ of $f(x)$ is defined by

$$\Delta^{\alpha} f(x) = (FW-h)^{\alpha} = \sum_{j=0}^{\infty} (-1)^j (j^{\alpha}) f(\gamma + (\alpha - k)h), \tag{4}$$

and its α-derivative is defined by

$$f^{\alpha}(x) = \lim_{h\to 0} \frac{\Delta^{\alpha} f(x)}{h^{\alpha}}. \tag{5}$$

For more details see [11,12].

Definition 2.3 (Modified Fractional Rieman-Lioville Derivative). *Let $f(x)$be the function that defined in definition 2.2, then*

(a) *If $f(x) = c$,where c is constant , then its α-derivative of order α is given by*

$$D_x^{\alpha} f(x) = \begin{cases} \frac{c}{\Gamma(1-\alpha)x^{\alpha}}, & \alpha \le 0 \\ 0, & 0 \end{cases}$$

(b) *If $f(x)$ is not constant, hence*

$$f(x) = f(0) + (f(x) - f(0))$$

With fractional derivative given by

$$f^{\alpha}(x) = D_x^{\alpha} f(0) + D_x^{\alpha} (f(x) - f(0)),$$

for $\alpha > 0$, we will put

$$D_x^{\alpha} (f(x) - f(0)) = D_x^{\alpha} f(x) = D_x \left(f^{\alpha-1}(x) \right).$$

If, $k < \alpha < k+1$, we will put

$$f^{\alpha}(x) = \left(f^{\alpha-k}(x) \right)^k, k \le \alpha \le k+1, k \ge 1.$$

Theorem 2.1. *The solution of fractional differential equation $dy = f(x)(dx)^{\alpha}, x > 0, y(0) = 0$, is defined by:*

$$\begin{aligned} y(x) &= \int_0^x f(u)(du)^{\alpha} \quad , y(0) = 0 \\ &= \alpha \int_0^x (x-u)^{\alpha-1} f(u)\, du \quad 0 < \alpha < 1, \end{aligned}$$

where the integration is taken with respect to $(dx)^{\alpha}$.

For more result see [13,14].

Definition 2.4 ([17]). *The Mittag-Leffler function is defined by*

$$E_{\alpha}(x) = \sum_{j=0}^{\infty} \frac{x^j}{\Gamma(\alpha j + 1)}, \ \mathrm{x} \in \mathbb{C}, \Re(\alpha) > 0. \tag{6}$$

Definition 2.5 ([10]). *The fractional Aboodh transfrom of function $f(x)$ is given by*

$$A_{\alpha}[f(x)] = K_{\alpha}(P) = \frac{1}{p} \int_0^{\infty} E_{\alpha}(-(px)^{\alpha}) f(x)(dx)^{\alpha}, p \in \mathbb{C}, x > 0. \tag{7}$$

Definition 2.6 ([17]). *The fractional double Laplace transfrom of function $f(x,y)$ is given by*

$$F_{\alpha}^2(p,q) = \int_0^{\infty} \int_0^{\infty} E_{\alpha}\left(-(px+qt)^{\alpha}\right) f(x,t)(dx)^{\alpha}(dt)^{\alpha}, \ \mathrm{p,q} \in \mathbb{C}. \tag{8}$$

3 Double Aboodh Transform of Fractional Order

The next definition characterizes the Double Aboodh transform of fractional order.

Definition 3.1. *Let $f(x,t)$ be a function where $x,t>0$, then the Double Aboodh transform of fractional order α is defined by*

$$A_\alpha^2\left(f(x,y)\right)=K_\alpha(p,q)=\tfrac{1}{pq}\int_0^\infty\int_0^\infty E_\alpha\left(-(px+qt)^\alpha\right)f(x,t)\,(dx)^\alpha\,(dt)^\alpha\,,$$
$$=\lim_{V,R\to\infty}\tfrac{1}{pq}\int_0^V\int_0^R E_\alpha\left(-(px+qt)^\alpha\right)f(x,t)\,(dx)^\alpha\,(dt)^\alpha\,,\ \ \mathrm{p,q}\in\mathbb{C}. \tag{9}$$

By using the property of Mittag-Leffler function (9) can be written as:

$$A_\alpha^2\left(f(x,y)\right)=K_\alpha(p,q)=\frac{1}{pq}\int_0^\infty\int_0^\infty E_\alpha\left(-(px)^\alpha\right)E_\alpha\left(-(qt)^\alpha\right)f(x,t)\,(dx)^\alpha\,(dt)^\alpha\,. \tag{10}$$

Remark 3.1. When $\alpha=1$,, Fractional double Aboodh transfrom (9) turns to Double Aboodh transform (1).

Remark 3.2. From definition (9), $K_\alpha(p,0)=K_\alpha(p),\ K_\alpha(0,q)=K_\alpha(q),$ where $K_\alpha(.)$, denote the fractional Aboodh transform given by equation (7).

4 Properties Of fractional double Aboodh transform

4.1 Linearity property

Theorem 4.1. *Let $f(x,t),\ g(x,t)$ be two functions, then:*

$$A_\alpha^2\left\{a\,f\,(x,t)+b\,g\,(x,t)\right\}=aA_\alpha^2\ \left\{f\,(x,t)\right\}+bA_\alpha^2\left\{g\,(x,t)\right\},$$

where a,b are constants.

Proof. By applying the definition of factional Aboodh transform, we can simply get the proof. □

4.2 Changing of scale

Theorem 4.2. *If $A_\alpha^2\left\{g\,(x,t)\right\}=k_\alpha\,(p,q)$, then*

$$A_\alpha^2\left\{g\,(ax,bt)\right\}=\frac{1}{a^\alpha b^\alpha}k_\alpha\left(\frac{p}{a},\frac{q}{b}\right),$$

where a,b are constants.

Proof. $A_\alpha^2\left\{g\,(ax,bt)\right\}=\frac{1}{pq}\int_0^\infty\int_0^\infty E_\alpha\left(-(px+qt)^\alpha\right)g\,(ax,bt)\,(dx)^\alpha\,(dt)^\alpha\,.$

By letting $u=ax,v=bt$ we get:

$$A_\alpha^2\left\{g\,(ax)\right\}=\frac{1}{a^\alpha b^\alpha}\frac{1}{pq}\int_0^\infty E_\alpha\left(-\left(\frac{p}{a}u+\frac{q}{b}v\right)^\alpha\right)g\,(u,v)\,(du)^\alpha\,(dv)^\alpha=\frac{1}{a^\alpha b^\alpha}k_\alpha\left(\frac{p}{a},\frac{q}{b}\right).$$

□

4.3 Shifting property

Theorem 4.3. *If $A_\alpha^2\left\{g\,(x,t)\right\}=k_\alpha\,(p,q)$, then $A_\alpha^2\left\{E_\alpha\,(-ax-bt)^\alpha\,g\,(x,t)\right\}=k_\alpha\,(p+a,q+b)$, where a,b are constants.*

Proof. We have

$$A_\alpha\left\{E_\alpha\left(-(ax+bt)^\alpha\right)g\,(x,y)\right\}=\frac{1}{pq}\int_0^\infty\int_0^\infty E_\alpha\left(-(px+qt)^\alpha\right)$$
$$E_\alpha\left(-(ax+bt)^\alpha\right)g\,(x,t)\,(dx)^\alpha\,(dt)^\alpha\,.$$

By using the following property of the Mittag-Leffler function,

$$E_\alpha\left(-(px+qt)^\alpha\right)E_\alpha\left(-(ax+by)^\alpha\right)\ =E_\alpha\left(-((p+a)x+(q+b)t))^\alpha\right)$$

we get:

$$A_\alpha^2\left\{E_\alpha\,(-ax-bt)^\alpha\,g\,(x,t)\right\}=\tfrac{1}{pq}\int_0^\infty E_\alpha\left(-((p+a)x+(q+b))\right)^\alpha g\,(x,t)\,(dx)^\alpha\,(dt)^\alpha$$
$$=k_\alpha\,(p+a,q+b)$$

□

4.4 Multiplication by $x^{\alpha}t^{\alpha}$

Theorem 4.4. *If* $A_{\alpha}^{2}\{g(x,t)\} = k_{\alpha}(p,q)$ *, then* $A_{\alpha}^{2}(x^{\alpha}t^{\alpha}g(x,t)) = \frac{1}{pq}D_{p}^{\alpha}D_{q}^{\alpha}(pqk_{\alpha}(p,q))$.

Proof. We have

$$\begin{aligned} A_{\alpha}^{2}(x^{\alpha}t^{\alpha}g(x,t)) &= \frac{1}{pq}\int_{0}^{\infty} E_{\alpha}(-(px+qt)^{\alpha})x^{\alpha}t^{\alpha}g(x,t)(dx)^{\alpha}(dt)^{\alpha} \\ &= \frac{1}{pq}\int_{0}^{\infty}\int_{0}^{\infty} D_{p}^{\alpha}D_{q}^{\alpha}[E_{\alpha}(-(px+qt)^{\alpha})]g(x,t)(dx)^{\alpha}(dt)^{\alpha} \\ &= \frac{1}{pq}D_{p}^{\alpha}D_{q}^{\alpha}\left[\int_{0}^{\infty}\int_{0}^{\infty} E_{\alpha}(-(px+qt)^{\alpha})(dx)^{\alpha}(dt)^{\alpha}\right] \\ &= \frac{1}{pq}D_{p}^{\alpha}D_{q}^{\alpha}(pq\,k_{\alpha}(p,q)). \end{aligned}$$

□

4.5 Fractional double Aboodh transform of some fractional partial derivatives

Theorem 4.5. *The fractional double Aboodh transform of fractional partial derivative respect to x is given by*

$$A_{\alpha}^{2}\left(\frac{\partial^{\alpha}}{\partial x^{\alpha}}g(x,t)\right) == p^{\alpha}K_{\alpha}(p,q) - \frac{(1+\alpha)}{p}K_{\alpha}(0,q).$$

Proof. By applying Fractional integration by part formula with respect to x, we get:

$$\begin{aligned} &\frac{1}{pq}\int_{0}^{\infty} E_{\alpha}(-(qt)^{\alpha})\left[(1+\alpha)g(x,t)E_{\alpha}(-(px)^{\alpha})|_{0}^{\infty} - \int_{0}^{\infty} D_{x}^{\alpha}E_{\alpha}(-(px)^{\alpha})g(x,t)(dx)^{\alpha}\right](dt)^{\alpha} \\ &= \frac{1}{pq}\int_{0}^{\infty} -(1+\alpha)g(0,t)(dt)^{\alpha} + p^{\alpha}\frac{1}{pq}\int_{0}^{\infty}\int_{0}^{\infty} E_{\alpha}(-(qt)^{\alpha})E_{\alpha}(-(px)^{\alpha})g(x,t)(dx)^{\alpha}(dt)^{\alpha} \\ &= -\frac{(1+\alpha)K_{\alpha}(0,q)}{p} + p^{\alpha}A_{\alpha}^{2}(g(x,t)) \\ &= p^{\alpha}K_{\alpha}(p,q) - \frac{(1+\alpha)}{p}K_{\alpha}(0,q). \end{aligned}$$

□

Theorem 4.6. *The fractional double Aboodh transform of fractional partial derivative respect to t is given by:*

$$A_{\alpha}^{2}\left(\frac{\partial^{\alpha}}{\partial t^{\alpha}}g(x,t)\right) = q^{\alpha}K_{\alpha}(p,q) - \frac{(1+\alpha)}{q}K_{\alpha}(p,0).$$

Proof. Similar to that of theorem 4.5. □

Theorem 4.7. *The fractional double Aboodh transform of a mixed fractional partial derivative is given by:*

$$A_{\alpha}^{2}\left[\frac{\partial^{2\alpha}}{\partial x^{\alpha}\partial t^{\alpha}}g(x,t)\right] = (pq)^{\alpha}K_{\alpha}(p,q) - \frac{p^{\alpha}}{q}(\alpha!)K_{\alpha}(p,0) - \frac{q^{\alpha}}{p}(\alpha!)K_{\alpha}(0,q) + \frac{1}{pq}(\alpha!)^{2}g(0,0)$$

Proof. Proof: Immediate by theorems 4.5 and 4.6. □

Remark 4.1. All results above are sutable for double Aboodh transform when $\alpha = 1$.

5 Convolution Theorem of Fractional Double Aboodh Transform

Proposition 5.1. *The double convolution of order* α *of functions* $f(x,t)$ *and* $g(x,t)$ *can be defined by the expression:*

$$(f(x,t) **_{\alpha} g(x,t)) = \int_{0}^{x}\int_{0}^{t} f(x-z,t-w)g(z,w)(dz)^{\alpha}(dw)^{\alpha}. \tag{11}$$

Therefore, the fractional Aboodh transform of (11) *is given by:*

$$A_{\alpha}^{2}\{f(x,t) **_{\alpha} g(x,t)\} = pqA_{\alpha}^{2}\{f(x,t)\}A_{\alpha}^{2}\{g(x,t)\}$$

Proof. We have

$$A_{\alpha}^{2}\{f(x,t)**_{\alpha}g(x,t)\}=\frac{1}{pq}\int_{0}^{\infty}\int_{0}^{\infty}E_{\alpha}\left(-(px+pt)^{\alpha}\right)\begin{bmatrix}\int_{0}^{x}\int_{0}^{t}f(x-z,t-w)\\ g(z,w)(dz)^{\alpha}(dw)^{\alpha}\end{bmatrix}(dx)^{\alpha}(dt)^{\alpha}\ (12)$$

By letting $u=x-z, v=t-w$and taking the limit from zero to infinity, (12) becomes:

$$\begin{aligned}&=\frac{1}{pq}\int_{0}^{\infty}\int_{0}^{\infty}\begin{bmatrix}E_{\alpha}\left(-(p(u+z))^{\alpha}\right)E_{\alpha}\left(-(q(v+w))^{\alpha}\right)\\ \left(\int_{0}^{\infty}\int_{0}^{\infty}f(u,v)g(z,w)(dz)^{\alpha}(dw)^{\alpha}\right)(du)^{\alpha}(dv)^{\alpha}\end{bmatrix}\\ &=\begin{bmatrix}\left(\frac{1}{pq}\int_{0}^{\infty}\int_{0}^{\infty}E_{\alpha}\left(-(pz)^{\alpha}\right)E_{\alpha}\left(-(qw)^{\alpha}\right)g(z,w)(dz)^{\alpha}(dw)^{\alpha}\right)\\ \times\left(\int_{0}^{\infty}\int_{0}^{\infty}E_{\alpha}\left(-(pu)^{\alpha}\right)E_{\alpha}\left(-(qv)^{\alpha}\right)f(u,v)(du)^{\alpha}(dv)^{\alpha}\right)\end{bmatrix}\\ &=pqA_{\alpha}^{2}\{f(x,t)\}A_{\alpha}^{2}\{g(x,t)\}\end{aligned}$$

□

6 Inversion formula of fractional double Aboodh transform

Definition 6.1 ([17]). *The Dirac's distripution $\delta_{\alpha}(x,t)$ of order α, where $\alpha\in(0,1)$is defined by:*

$$\int_{\Re}\int_{\Re}f(x,t)\,\delta_{\alpha}(x-a,t-b)(dx)^{\alpha}(dt)^{\alpha}=\alpha^{2}f(a,b). \qquad (12)$$

Example 6.1. *The fractional double Aboodh transform of $\delta_{\alpha}(x-a,t-b)$ can be calculated as following:*

$$\begin{aligned}A_{\alpha}^{2}\{\delta_{\alpha}(x-a,t-b)\}&=\frac{1}{p}\int_{0}^{\infty}\int_{0}^{\infty}E_{\alpha}\left(-(px+qt)^{\alpha}\right)\delta_{\alpha}(x-a,t-b)(dx)^{\alpha}(dt)^{\alpha}\\ &=\frac{\alpha}{pq}E_{\alpha}\left(-(pa+qb)^{\alpha}\right).\end{aligned}$$

In particular,

$$A_{\alpha}^{2}\{\delta_{\alpha}(x,t)\}=\frac{\alpha}{pq}.$$

The following Lemma clarify the relation between $\delta_{\alpha}(x,t)$ and $E_{\alpha}(x+t)^{\alpha}$, this Lemma helps us to prove inversion theorem that we will consider later.

Lemma 6.1 ([17]). *The following formula holds*

$$\frac{\alpha^{2}}{(\mu_{\alpha})^{2\alpha}}\int_{\Re}\int_{\Re}E_{\alpha}\left(i(-ux)^{\alpha}\right)E_{\alpha}\left(i(-wt)^{\alpha}\right)(du)^{\alpha}(dw)^{\alpha}=\delta_{\alpha}(x,t). \qquad (13)$$

Whereμ_{α}, satisfy$E_{\alpha}\left(i(\mu_{\alpha})^{\alpha}\right)=1$, and called the period of the complex-valued Mittag-leffer function .

Proof. We test that (13) is consistent with

$$\int_{\Re}\int_{\Re}E_{\alpha}\left(i(ux)^{\alpha}\right)E_{\alpha}\left(i(wt)^{\alpha}\right)\delta_{\alpha}(x,t)(dx)^{\alpha}(dt)^{\alpha}=\alpha^{2}. \qquad (14)$$

By substituting (13), in (14), we get:

$$\begin{aligned}\alpha^{2}&=\int_{\Re}\int_{\Re}(dx)^{\alpha}(dt)^{\alpha}E_{\alpha}\left(i(ux)^{\alpha}\right)E_{\alpha}\left(i(wt)^{\alpha}\right)\frac{\alpha^{2}}{(\mu_{\alpha})^{2\alpha}}\int_{\Re}\int_{\Re}E_{\alpha}\left(i(-vx)^{\alpha}\right)E_{\alpha}\left(i(-st)^{\alpha}\right)(dv)^{\alpha}(ds)^{\alpha}\\ &=\int_{\Re}\int_{\Re}(dx)^{\alpha}(dt)^{\alpha}\frac{\alpha^{2}}{(\mu_{\alpha})^{2\alpha}}\int_{\Re}\int_{\Re}E_{\alpha}\left(i((u-v)x)^{\alpha}\right)E_{\alpha}\left(i((w-s)t)^{\alpha}\right)(dv)^{\alpha}(ds)^{\alpha}\\ &=\int_{\Re}\int_{\Re}(dx)^{\alpha}(dt)^{\alpha}\frac{\alpha^{2}}{(\mu_{\alpha})^{2\alpha}}\int_{\Re}\int_{\Re}E_{\alpha}\left(i(xp)^{\alpha}\right)E_{\alpha}\left(i(qt)^{\alpha}\right)(dp)^{\alpha}(dq)^{\alpha}\\ &=\int_{\Re}\int_{\Re}\delta_{\alpha}(x,t)(dx)^{\alpha}(dt)^{\alpha}\\ &=\alpha^{2}.\end{aligned}$$

□

7 Inversion Theorem of Fractional Double Aboodh Transform

Theorem 7.1. *The inverse of the Fractional double Aboodh transform (9) is defined as:*

$$f(x,t) = \frac{1}{(\mu_\alpha)^{2\alpha}} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} pq\, E_\alpha (px)^\alpha E_\alpha (qt)^\alpha k(p,q) (dp)^\alpha (dq)^\alpha \tag{15}$$

Proof. By subsituting (15) into (9) and using (13) and (14) we get:

$$\begin{aligned} f(x,t) &= \tfrac{1}{(\mu_\alpha)^{2\alpha}} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} pq\, E_\alpha (px)^\alpha E_\alpha (qt)^\alpha (dp)^\alpha (dq)^\alpha \\ &\quad \tfrac{1}{pq} \int_0^\infty \int_0^\infty E_\alpha (-pu)^\alpha E_\alpha (-qv)^\alpha f(u,v) (du)^\alpha (dv)^\alpha \\ &= \tfrac{1}{(\mu_\alpha)^{2\alpha}} \int_0^\infty \int_0^\infty f(u,v) (du)^\alpha (dv)^\alpha \\ &\quad \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} E_\alpha (p(x-u))^\alpha E_\alpha (q(t-v))^\alpha (dp)^\alpha (dq)^\alpha \\ &= \tfrac{1}{(\mu_\alpha)^{2\alpha}} \int_0^\infty \int_0^\infty \tfrac{(\mu_\alpha)^{2\alpha}}{\alpha^2} f(u,v) \delta_\alpha (u-x, v-t) (du)^\alpha (dv)^\alpha \\ &= \int_0^\infty \int_0^\infty \tfrac{1}{\alpha^2} f(u,v) \delta_\alpha (u-x, v-t) (du)^\alpha (dv)^\alpha \\ &= \tfrac{1}{\alpha^2} . \alpha^2 f(x,t) = f(x,t). \end{aligned}$$

□

8 Conclusion

In this paper, we introduce the fractional double Aboodh transform and its inverse. Several theorems and properties of fractional double Aboodh transform have been discussed and proved. Our results are consistent with Aboodh transform when $\alpha = 1$.

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